## UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS International General Certificate of Secondary Education

## MATHEMATICS

0580/04
0581/04
Paper 4 (Extended)

Additional Materials:

> Answer Booklet/Paper Electronic calculator Geometrical instruments Graph paper (3 sheets) Mathematical tables (optional) Tracing paper (optional)

May/June 2005
2 hours 30 minutes

1 Hassan sells fruit and vegetables at the market.
(a) The mass of fruit and vegetables he sells is in the ratio

$$
\text { fruit : vegetables }=5: 7 .
$$

Hassan sells 1.33 tonnes of vegetables.
How many kilograms of fruit does he sell?
(b) The amount of money Hassan receives from selling fruit and vegetables is in the ratio
fruit : vegetables $=9: 8$.
Hassan receives a total of $\$ 765$ from selling fruit and vegetables.
Calculate how much Hassan receives from selling fruit.
(c) Calculate the average price of Hassan's fruit, in dollars per kilogram.
(d) (i) Hassan sells oranges for $\$ 0.35$ per kilogram.

He reduces this price by $40 \%$.
Calculate the new price per kilogram.
(ii) The price of $\$ 0.35$ per kilogram of oranges is an increase of $25 \%$ on the previous day's price.

Calculate the previous day's price.

## 2 Answer the whole of this question on a new page.



NOT TO
SCALE

The diagram shows a trapezium $A B C D$.
$A B=12 \mathrm{~cm}, D C=9 \mathrm{~cm}$ and the perpendicular distance between these parallel sides is 7 cm .
$A D=B C$.
(a) Approximately halfway down your page, draw a line $A B$ of length 12 cm .
(b) Using a straight edge and compasses only, construct the perpendicular bisector of $A B$.
(c) Complete an accurate drawing of the trapezium $A B C D$.
(d) Measure angle $A B C$, giving your answer correct to the nearest degree.
(e) Use trigonometry to calculate angle $A B C$.

Show all your working and give your answer correct to 1 decimal place.
(f) On your diagram,
(i) draw the locus of points inside the trapezium which are 5 cm from $D$,
(ii) using a straight edge and compasses only, construct the locus of points equidistant from $D A$ and from $D C$,
(iii) shade the region inside the trapezium containing points which are less than 5 cm from $D$ and nearer to $D A$ than to $D C$.

(a) Describe fully the single transformation which maps
(i) triangle $X$ onto triangle $P$,
(ii) triangle $X$ onto triangle $Q$,
(iii) triangle $X$ onto triangle $R$,
(iv) triangle $X$ onto triangle $S$.
(b) Find the 2 by 2 matrix which represents the transformation that maps
(i) triangle $X$ onto triangle $Q$,
(ii) triangle $X$ onto triangle $S$.

## 4 Answer the whole of this question on a sheet of graph paper.

The table gives values of $\mathrm{f}(x)=2^{x}$, for $-2 \leqslant x \leqslant 4$.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $p$ | 0.5 | $q$ | 2 | 4 | $r$ | 16 |

(a) Find the values of $p, q$ and $r$.
(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 1 cm to 1 unit on the $y$-axis, draw the graph of $y=\mathrm{f}(x)$ for $-2 \leqslant x \leqslant 4$.
(c) Use your graph to solve the equation $2^{x}=7$.
(d) What value does $\mathrm{f}(x)$ approach as $x$ decreases?
(e) By drawing a tangent, estimate the gradient of the graph of $y=\mathrm{f}(x)$ when $x=1.5$.
(f) On the same grid draw the graph of $y=2 x+1$ for $0 \leqslant x \leqslant 4$.
(g) Use your graph to find the non-integer solution of $2^{x}=2 x+1$.

5


NOT TO
SCALE
$O A B C D E$ is a regular hexagon.
With $O$ as origin the position vector of $C$ is $\mathbf{c}$ and the position vector of $D$ is $\mathbf{d}$.
(a) Find, in terms of $\mathbf{c}$ and $\mathbf{d}$,
(i) $\overrightarrow{D C}$,
(ii) $\overrightarrow{O E}$,
(iii) the position vector of $B$.
(b) The sides of the hexagon are each of length 8 cm .

Calculate
(i) the size of angle $A B C$,
(ii) the area of triangle $A B C$,
(iii) the length of the straight line $A C$,
(iv) the area of the hexagon.

## NOT TO SCALE



The diagram shows a pencil of length 18 cm .
It is made from a cylinder and a cone.
The cylinder has diameter 0.7 cm and length 16.5 cm .
The cone has diameter 0.7 cm and length 1.5 cm .
(a) Calculate the volume of the pencil.
[The volume, $V$, of a cone of radius $r$ and height $h$ is given by $V=\frac{1}{3} \pi r^{2} h$.]
(b)


Twelve of these pencils just fit into a rectangular box of length 18 cm , width $w \mathrm{~cm}$ and height $x \mathrm{~cm}$. The pencils are in 2 rows of 6 as shown in the diagram.
(i) Write down the values of $w$ and $x$.
(ii) Calculate the volume of the box.
(iii) Calculate the percentage of the volume of the box occupied by the pencils.
(c) Showing all your working, calculate
(i) the slant height, $l$, of the cone,
(ii) the total surface area of one pencil, giving your answer correct to 3 significant figures.
[The curved surface area, $A$, of a cone of radius $r$ and slant height $l$ is given by $A=\pi r l$.]

7 The speeds ( $v$ kilometres/hour) of 150 cars passing a $50 \mathrm{~km} / \mathrm{h}$ speed limit sign are recorded. A cumulative frequency curve to show the results is drawn below.

(a) Use the graph to find
(i) the median speed,
(ii) the inter-quartile range of the speeds,
(iii) the number of cars travelling with speeds of more than $50 \mathrm{~km} / \mathrm{h}$.
(b) A frequency table showing the speeds of the cars is

| Speed $(v \mathrm{~km} / \mathrm{h})$ | $30<v \leqslant 35$ | $35<v \leqslant 40$ | $40<v \leqslant 45$ | $45<v \leqslant 50$ | $50<v \leqslant 55$ | $55<v \leqslant 60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 10 | 17 | 33 | 42 | $n$ | 16 |

(i) Find the value of $n$.
(ii) Calculate an estimate of the mean speed.
(c) Answer this part of this question on a sheet of graph paper.

Another frequency table for the same speeds is

| Speed $(v \mathrm{~km} / \mathrm{h})$ | $30<v \leqslant 40$ | $40<v \leqslant 55$ | $55<v \leqslant 60$ |
| :---: | :---: | :---: | :---: |
| Frequency | 27 | 107 | 16 |

Draw an accurate histogram to show this information.
Use 2 cm to represent 5 units on the speed axis and 1 cm to represent 1 unit on the frequency density axis (so that $1 \mathrm{~cm}^{2}$ represents 2.5 cars).

8

$$
\mathrm{f}(x)=x^{2}-4 x+3 \quad \text { and } \quad \mathrm{g}(x)=2 x-1
$$

(a) Solve $\mathrm{f}(x)=0$.
(b) Find $\mathrm{g}^{-1}(x)$.
(c) Solve $\mathrm{f}(x)=\mathrm{g}(x)$, giving your answers correct to 2 decimal places.
(d) Find the value of $\mathrm{gf}(-2)$.
(e) Find $\mathrm{fg}(x)$. Simplify your answer.

## 9 Answer the whole of this question on a sheet of graph paper.

A taxi company has "SUPER" taxis and "MINI" taxis.
One morning a group of 45 people needs taxis.
For this group the taxi company uses $x$ "SUPER" taxis and $y$ "MINI" taxis.
A "SUPER" taxi can carry 5 passengers and a "MINI" taxi can carry 3 passengers.
So $5 x+3 y \geqslant 45$.
(a) The taxi company has 12 taxis.

Write down another inequality in $x$ and $y$ to show this information.
(b) The taxi company always uses at least 4 "MINI" taxis.

Write down an inequality in $y$ to show this information.
(c) Draw $x$ and $y$ axes from 0 to 15 using 1 cm to represent 1 unit on each axis.
(d) Draw three lines on your graph to show the inequality $5 x+3 y \geqslant 45$ and the inequalities from parts
(a) and (b).

Shade the unwanted regions.
(e) The cost to the taxi company of using a "SUPER" taxi is $\$ 20$ and the cost of using a "MINI" taxi is $\$ 10$.
The taxi company wants to find the cheapest way of providing "SUPER" and "MINI" taxis for this group of people.
Find the two ways in which this can be done.
(f) The taxi company decides to use 11 taxis for this group.
(i) The taxi company charges $\$ 30$ for the use of each "SUPER" taxi and $\$ 16$ for the use of each "MINI" taxi.
Find the two possible total charges.
(ii) Find the largest possible profit the company can make, using 11 taxis.

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